**Real Analysis Exchange**
Summer Symposium XXXX, 2016, pp. 1–2

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# ON SMALL DIFFERENCES AND LARGE SUMS OF SETS

Let $A := \{2, 6, 7, 9, 10, 11\} \subset \mathbb{Z}_{12}$. R. Zduńczyk noticed that the algebraic sum $A + A$ is equal to the $\mathbb{Z}_{12}$ while 6 does not belong to $A - A$. Making use of this observation we show the existence of

- compact subsets of $\mathbb{R}$ with similar properties (attractors of affine iterated function systems);
- $\sigma$-ideals of subsets of compact groups such that sets not belonging to the $\sigma$-ideal (or closed sets which do not belong to the $\sigma$-ideal) have "similar properties.

More precisely we prove:

**Theorem 1.** *There is a compact set $X \subset \mathbb{R}$ such that $int(X + X) \neq \varnothing$ and $X - X$ is a null set. [2]*

Suppose that $G$ is an abelian topological group and $\mathcal{J}$ is an ideal on a group $G$. We say that $\mathcal{J}$ has the closed $(1, 1)$-Steinhaus property ($(1, -1)$-Steinhaus property) if for any closed subsets $X_1, X_2 \notin \mathcal{J}$ the set $X_1 + X_2$ ($X_1 - X_2$, respectively) is not nowhere dense in $G$ ([1]).

**Theorem 2.** *There exists a compact abelian topological group $G$ and an ideal $\mathcal{J}$ on $G$ such that $\mathcal{J}$ has the closed $(1, 1)$-Steinhaus property and does not have the closed $(1, -1)$-Steinhaus property. [2]*

Mathematical Reviews subject classification: Primary: 20K01; Secondary: 28A80, 54H11
Key words: group, nonempty interior, closed Steinhaus property, attractor od IFS

**Theorem 1 ([2])**: There is a compact set $X \subset \mathbb{R}$ such that $\operatorname{int}(X+X) \neq \varnothing$ and $X - X$ is a null set.

Suppose that $G$ is an abelian topological group and $\mathcal{J}$ is an ideal on a group $G$. We say that $\mathcal{J}$ has the closed $(1,1)$-Steinhaus property ($(1,-1)$-Steinhaus property) if for any closed subsets $X_1, X_2 \notin \mathcal{J}$ the set $X_1 + X_2$ ($X_1 - X_2$, respectively) is not nowhere dense in $G$ ([1]).

**Theorem 2 ([2])**: There exists a compact abelian topological group $G$ and an ideal $\mathcal{J}$ on $G$ such that $\mathcal{J}$ has the closed $(1,1)$-Steinhaus property and does not have the closed $(1,-1)$-Steinhaus property.

## References

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[2] A. Bartoszewicz, M. Filipczak, *Remarks on sets with small differences and large sums*, submitted.