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SETS CONTAINING THE SKELETON OF A UNIT CUBE CENTERED AT EVERY POINT OF $\mathbb{R}^n$

We study the following question: How small can a set $B \subset \mathbb{R}^n$ be if it contains the k-skeleton of an n-dimensional (rotated) unit cube centered at every point of $\mathbb{R}^n$? As one result we show that for any integers $0 \leq k < n$, there exists such a Borel set of Hausdorff dimension $k+1$. In fact, we have even more: for any integers $0 \leq k < n$, there exists a Borel set B of Hausdorff dimension $k+1$ such that for all $x \in \mathbb{R}^n$ and all $r > 0$, B contains the k-skeleton of a (rotated) cube of size r centered at x. In the proofs we use typical constructions in the Baire category sense. We also have the following: If a set $B \subset \mathbb{R}^n$ contains the boundary ($(n-1)$-skeleton) of a unit cube centered at every point of $\mathbb{R}^n$, then the Hausdorff dimension of B must be n, and there exists a Borel set of Lebesgue measure zero having this property. The presented results are very recent, obtained in a joint work with A. Chang, M. Csörnyei, and T. Keleti.

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