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A UNIFIED APPROACH TO LIMIT THEOREMS IN NONLINEAR INTEGRATION THEORY

In 1974 Sugeno introduced the notion of fuzzy measure and fuzzy integral, which are now generalized to the nonadditive measure and the Sugeno integral, to evaluate nonadditive or nonlinear quality in systems engineering. A nonadditive measure is a set function that vanishes at the empty set and is monotonely increasing. This type of set function, in conjunction with several types of nonlinear integrals, is closely related to the theory of capacities and imprecise probabilities and has been extensively studied with applications to decision theory under uncertainty, game theory, data mining, and some economic topics under Knightian uncertainty and others.

In order to put nonlinear integrals into practical use, it is inevitable to formulate some limit theorems assuring that the limit of the integrals of a sequence of functions is the integral of the limit function. Many attempts have thus been made to formulate such limit theorems for nonlinear integrals such as the Choquet [1, 6], the Šipoš [8], the Sugeno [9], and the Shilkret integral [7, 10]. However, to the best of knowledge, there is no unified approach to limit theorems in literature that are simultaneously applicable to both the Lebesgue integral as a linear integral and the Choquet, the Šipoš, the Sugeno, and the Shilkret integral as nonlinear integrals.

The purpose of the talk is to present a unified approach to limit theorems for nonlinear integrals. A nonlinear integral may be viewed as a nonlinear functional $I: \mathcal{M}(X) \times \mathcal{F}^+(X) \rightarrow [0, \infty]$, where X is a non-empty set, \mathcal{A} is a field of subsets of X , $\mathcal{M}(X)$ is the set of all nonadditive measures on (X, \mathcal{A}) , and $\mathcal{F}^+(X)$ is the set of all \mathcal{A} -measurable non-negative functions on X . We thus formulate our general type of limit theorem for such a functional. In

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particular, we report that the famous limit theorems for nonlinear integrals such as the bounded/monotone/pointwise convergence theorems follow from our limit theorems for functionals. The key tool is a perturbation of functional that manages not only the monotonicity of the functional but also the small change of the value $I(\mu, f)$ arising as a result of adding small amounts to the measure μ and the function f in the domain of the functional.

This talk contains selected results from [2, 3, 4, 5].

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