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# TSIRELSON-LIKE SPACES AND COMPLEXITY OF CLASSES OF BANACH SPACES

During the last two decades, it turned out that descriptive set theory provides a fruitful approach to several questions in separable Banach space theory. A particular and generally still not well understood question is the question of the descriptive complexity of a given class of separable Banach spaces. In the present work, we introduce a new approach to complexity problems in Banach space theory which is based on a fundamental example of Tsirelson.

Let SB denote the standard Borel space of all separable Banach spaces. That is, SB is the set of all closed linear subspaces of $C(2^{\mathbb{N}})$ equipped with the Effros Borel structure, defined as the $\sigma$-algebra generated by the sets

$$\{X \in \mathrm{SB} : X \cap U \neq \emptyset\},$$

where $U$ varies over open subsets of $C(2^{\mathbb{N}})$.

Employing a construction of Tsirelson-like spaces due to Argyros and Deliyanni, we establish the following results. (1) The set of all $X \in \mathrm{SB}$ which can be embedded isomorphically into $c_0$ is a complete analytic set. (2) The set of all $X \in \mathrm{SB}$ with the Schur property is a $\mathbf{\Pi}^1_2$-complete set.