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RECOVERING A COMPACT HAUSDORFF SPACE X FROM THE COMPATIBILITY ORDERING ON $C(X)$

Let X and Y be compact Hausdorff spaces. Let $f, g \in C(X)$ where $C(X)$ denotes the space of continuous functions on X . We say that g dominates f in the compatibility ordering if g coincides with f on the support of f . Our main result states that X and Y are homeomorphic if and only if there exists a compatibility isomorphism $T : C(X) \rightarrow C(Y)$. We derive several classical theorems of functional analysis as easy corollaries to our result:

If X and Y are compact Hausdorff spaces, we obtain that they are homeomorphic provided that there exists a bijection $T : C(X) \rightarrow C(Y)$ satisfying one of the following conditions:

- T is a ring isomorphism (Gelfand–Kolmogorov);
- T is multiplicative (Milgram);
- T the ordinary pointwise ordering (Kaplansky);
- T is linear, and $Tf \cdot Tg = 0$ whenever $f \cdot g = 0$ (Jarosz).

Moreover, we obtain an automatic continuity result: Any compatibility isomorphism is norm-continuous provided that X satisfies certain topological conditions. On the other hand, in other compact Hausdorff spaces we show the existence of discontinuous compatibility automorphisms of $C(X)$.

Key words: compact Hausdorff space, homeomorphism, continuous function, compatibility ordering, order isomorphism

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