Equilateral weights on subsets of $\mathbb{R}^n$

Emmanuel Chetcuti
University of Malta, Malta

A group-valued function f defined on a subset D of $\mathbb{R}^n$ is called an *equilateral weight* if the sum of the function values taken at the vertices of any (full-dimensional) regular simplex contained in D is the same; this constant is called the *weight* of the function. By an elementary argument, it easily follows that when $D = \mathbb{R}^n$, any equilateral weight must be constant. However, when restricting the domain D, there may exist appropriate functions which are not constant.

The aim of this presentation is to describe equilateral weights for a number of subsets of $\mathbb{R}^n$.

1. When D equals the sphere in $\mathbb{R}^n$ with radius $1/\sqrt{2}$, denoted by $S^n_{1/\sqrt{2}}$, every full-dimensional regular simplex corresponds to an orthogonal bases of $\mathbb{R}^n$ and the complete characterization of $\mathbb{R}$-valued equilateral weights on $S^n_{1/\sqrt{2}}$ is precisely the formulation of the celebrated Gleason Theorem [**?**].
2. Elementary but elegant combinatorial arguments show that the situation changes drastically when for D we take $S^n_{1/\sqrt{2}} \cap \mathbb{Q}^n$ [**?**].
3. Surprisingly, on the closed unit ball B^n of $\mathbb{R}^n$, there are no non-trivial equilateral weights [**?**].
4. Finally, we shall discuss $\mathbb{Z}_2$-valued weights on $S^n_{1/\sqrt{2}}$ giving an answer for $n \geqslant 4$ and state the problem left unsolved for the case when $n = 3$ [**?**].

Bibliography

[1] E. Chetcuti and J. Muscat, *Equilateral weights on the unit ball of* $\mathbb{R}^n$, Real Anal. Exchange **40** (2015), 37–51.

[2] A. M. Gleason, *Measures on the closed subspaces of a Hilbert space*, J. Math. Mech. **6** (1957), 885–893.

[3] C. D. Godsil and J. Zaks, *Coloring the sphere*, Combinatorics and Optimization, University of Waterloo research report CORA 88-12, 1988.

[4] M. Navara and P. Pták, *For* $n \geqslant 5$ *there is no nontrivial* $\mathbb{Z}_2$*-measure on* $L(\mathbb{R}^n)$, Internat. J. Theoret. Phys. **43** (2004), 1595–1598.