The Vitali convergence theorem for distribution-based nonlinear integrals

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The Lebesgue integral is used for aggregating an infinite number of inputs into a single output value and is continuous with respect to inputs by the Lebesgue convergence theorem. This continuity is a guarantee for certain robustness and consistency in the aggregation process.

The Choquet, the Sugeno, and the Shilkret integrals may be considered as nonlinear aggregation functionals $I\colon \mathcal{M}(X) \times \mathcal{F}^+(X) \to [0,\infty]$, where $\mathcal{M}(X)$ is the set of all nonadditive measures $\mu\colon \mathcal{A} \to [0,\infty]$ on a measurable space $(X,\mathcal{A})$ and $\mathcal{F}^+(X)$ is the set of all $\mathcal{A}$-measurable functions $f\colon X \to [0,\infty]$. For those functionals, their continuity corresponds to the convergence theorem of integrals, which means that the limit of the integrals of a sequence of functions is the integral of the limit function. Thus many attempts have been made to formulate the monotone, the bounded, and the dominated convergence theorems for the Choquet, the Sugeno, and the Shilkret integrals, all of which are determined through the μ-decreasing distribution function $G_\mu(t) := \mu(\{f \geqslant t\})$.

The purpose of this talk is to present the Vitali convergence theorem for such distribution-based nonlinear integrals. A key ingredient is a perturbation of functional that manages the change in the functional value $I(\mu,f)$ when the integrand is slightly shifted from f to $f+\varepsilon$ and the μ-decreasing distribution function is slightly shifted from $G_\mu(f)$ to $G_\mu(f)+\delta$.
