Structured population model with diffusion in structure space

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A structured population model is described and analyzed, in which individual dynamics may be stochastic. The model consists of a PDE of advection-diffusion type in the structure variable. The population may represent, for example, the density of infected individuals structured by pathogen density x, $x \geqslant 0$. The individuals with density $x = 0$ are not infected, but rather susceptible or recovered. Their dynamics is described by an ODE with a source term that is the exact flux from the diffusion and advection as $x \to 0^+$. Infection/reinfection is then modeled by distributing some fraction of these individuals into the infected class, but distributed in the structure variable through a probability density function. Existence of a global-in-time solution is proven, as well as a classical bifurcation result about equilibrium solutions: a net reproduction number R_0 is defined that separates the case of only the trivial equilibrium existing when $R_0 < 1$ from the existence of another —notrivial— equilibrium when $R_0 > 1$. Numerical simulation results are provided to show the stabilization towards the positive equilibrium when $R_0 > 1$ and towards the trivial one when $R_0 < 1$, result that is not proven analytically. Simulations are also provided to show the Allee effect that helps boost population sizes at low densities.
