U- and V-sets for multiple Vilenkin series

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Let $\{f_n\}$ be an orthonormal system in a Hilbert space $L_2(X)$. Then a set $A \subset X$ is called a *$\mathcal{V}$-set* for the system $\{f_n\}$ if convergence of a series $\sum_n a_n f_n(x)$ to a finite summable function f on $X \setminus A$ implies that this series is the Fourier series of f. Setting $f = 0$ on $X \setminus A$, we get the definition of a *$\mathcal{U}$-set* for the system $\{f_n\}$. Each $\mathcal{V}$-set is evidently a $\mathcal{U}$-set.

We study analysis on Vilenkin groups G, i.e., on zero-dimensional second-countable compact commutative groups (see [**?**]). The elements of the dual group of G form an orthonormal system $\{f_n\}$ in $L_2(G)$.

Harris proved [**?**] that any closed, measure zero subgroup of a Vilenkin group is a $\mathcal{U}$-set. Grubb found another examples of $\mathcal{U}$-sets and de-facto proved that any closed $\mathcal{U}$-set is a $\mathcal{V}$-set (see, for example, [**?**, **?**]). In [**?**] some category properties of $\mathcal{U}$-sets are established.

In the multidimensional case, examples of countable $\mathcal{U}$-sets for square convergence are constructed in [**?**]. We introduce a multidimensional analog of Dirichlet sets in the product of Vilenkin groups and prove that all translations of such sets are $\mathcal{V}$-sets and therefore $\mathcal{U}$-sets. The full sequence of rectangular partial sums and restricted rectangular convergence are considered. The main tool of our investigation is quasi-measures and the concept of Γ-continuity of ones.

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