On rearranged multiple Haar series

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Almost 150 years ago Cantor proved that *if a trigonometric series* $TS = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$ *everywhere converges to zero then* TS *is the trivial series.* We say that *uniqueness holds* for a system of functions and for some summation method if the analog of the Cantor theorem is true.

Consider the multivariate Haar system $\{\prod_{i=1}^{d} H_{n_i}(x_i)\}$. Skvortsov proved [**?**] that uniqueness holds for rectangular convergence. In contrast, Plotnikov established that uniqueness does not hold for square convergence. Moreover, for λ-convergent (= convergent over λ-regular rectangles) double Haar series, uniqueness does not hold if $\lambda < \sqrt{2}/2$ but holds if $\lambda > \sqrt{2}/2$ (see [**?**]).

For some rearrangements of the system $\{H_n(x) H_m(y)\}$, non-trivial everywhere λ-convergent series exist even if $\lambda < 2$ (see [**?**]). The constant 2 is the best if we consider some natural class of rearrangements. The following is true.

Theorem 1. *Let* T *be any permutation of the set* $\{0, 1, \ldots\}$ *such that* T *preserves all dyadic blocks* $\{2^{k-1}, \ldots, 2^k - 1\}$. *Then uniqueness holds for the system* $\left\{H_{T(n)}(x) H_{T(m)}(y)\right\}_{n,m=0}^{\infty}$ *under* λ*-convergence whenever* $\lambda \geqslant 2$.

In conclusion, we discuss some uniqueness problems concerning multiple series with respect to other systems of functions. In the case of rectangle convergence, uniqueness holds for the trigonometric system [**?**], for the Walsh system [**?**], and for the Franklin system [**?**]. Uniqueness also holds for spherical convergent multiple trigonometric series [**?**]. But the cases of square convergence and λ-convergence lead to open questions, even in dimension 2.

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