On products of nuclear operators

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Let $0 < s \leqslant 1$. An operator $T : X \to Y$ in Banach spaces is called s-nuclear (nuclear if $s = 1$) if it admits a representation of the kind $Tx = \sum_{k=1}^{\infty} \langle x'_k, x \rangle y_k$ for $x \in X$, where $(x'_k) \subset X^*, (y_k) \subset Y$, $\sum_k ||x'_k||^s \, ||y_k||^s < \infty$. We use the notation $N_s(X, Y)$ for the linear space of such operators. We denote by S_p $(0 < p < \infty)$ the von Neumann-Schatten class of operators in Hilbert spaces.

We say that an operator T *can be factored through an operator* from S_p, if there exist a Hilbert space H and the operators $A \in L(X, H)$, $U \in S_p(H)$ and $B \in L(H, Y)$, such that $T = BUA$.

The following result was inspired by a question of B. S. Mityagin: Is it true that a product of two nuclear operators can be factored through an operator from S_1?

Theorem. *If $X_1, \ldots, X_{n+1}$ are Banach spaces, $s_k \in (0, 1]$ and $T_k \in N_{s_k}(X_k, X_{k+1})$ for $k = 1, 2, \ldots, n$, then the product $T := T_n T_{n-1} \cdots T_1$ can be factored through an operator from S_r, where $1/r = 1/s_1 + 1/s_2 + \cdots + 1/s_n - (n + 1)/2$.*

We will discuss the sharpness of this result. For example, we have:

1) If an operator T in a Banach space is nuclear and $m > 1$, then T^m can be factored through an operator from S_r, where $r = 2/(m - 1)$.

2) There exists a nuclear operator T in the space $C[0, 1]$ (or in the space $L_1[0, 1]$) such that for any $m > 1$ and $r < 2/(m - 1)$ the operator T^m can not be factored through an operator from S_r.