

c -Removable sets: Old and new results

Martin Rmoutil (speaker), Dušan Pokorný
Charles University, Prague, Czech Republic

This work is still in progress. We study subsets of Euclidean spaces that are negligible from the point of view of convexity of functions (the “ c ” in c -removability comes from “convexity”). More precisely, a closed set $F \in \mathbb{R}^d$ is said to be c -removable if the following is satisfied: Whenever a continuous function $f : \mathbb{R}^d \rightarrow \mathbb{R}$ is locally convex on the complement of F , it is convex on the whole \mathbb{R}^d .

Five years ago, at the 37th Summer Symposium in Real Analysis (although that particular one was technically a “Winter Symposium” as it took place in the Southern Hemisphere), I presented joint results with Dušan Pokorný disproving a conjecture by Jacek Tabor and Józef Tabor that c -removability is characterized by *interval thinness*, a notion that they introduced, which means that the set is essentially transparent in all directions: We found examples of sets which are c -removable, yet not intervally thin (one such example we call the *Holey Devil’s Staircase*). We also found many examples of non- c -removable discontinua.

However, the question remained open of the existence of a nontrivial c -removable *continuum*. We now have such examples along with other new results, providing a better understanding of the notion of c -removability.
