Approximation by multivariate quasi-projection operators

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Quasi-projection operators

$$Q_j(f;\widetilde{\varphi},\varphi)=|\det M|^j\sum_{k\in\mathbb{Z}}\langle f,\widetilde{\varphi}(M^j\cdot-k)\rangle\varphi(M^j\cdot-k),$$

where M is a matrix dilation, is studied for a class of band-limited functions φ and different classes of funcions/distributions $\widetilde{\varphi}$. An important special case is so-called Kantorovich-Kotelnikov type operators, where $\widetilde{\varphi}$ is a summable function. The L_p-rate of convergence for such operators is given in terms of the classical moduli of smoothness. Several examples of the Kantorovich-Kotelnikov operators generated by the multivariate sinc-function and the linear combinations of its translations are provided. Similar estimates in the weighted L_p spaces are obtained under additional assunption of some smootness of the Fourier transform of φ. This allows to estimate the error for reconstruction of signals (approximated functions) whose decay is not enough to be in L_p. Another special case is the sampling expansions, i.e., the case, where $\widetilde{\varphi}$ is the the Dirac delta-function. Approximation order in L_p-norm, $p\geqslant 2$, is investigated for such operators. These results are extended to the weighted L_p-norm for the Muckenhoup weights.