Some problems in harmonic analysis on compact zero-dimensional groups (non-abelian case)

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Here we extend some of our previous results obtained in [?] and [?] to the case of non-abelian groups.

Let G be a compact, 0-dimensional, metric group, *not necessarily abelian*, and let $\{G_n\}$ be a strictly decreasing sequence of open normal subgroups forming a neighborhood base at the identity. Let Σ denote the dual object of G, i. e., the set of equivalence classes of irreducible representations of G. If $\sigma \in \Sigma$, we pick a irreducible representation U^σ in the equivalence class σ. Let the representation U^σ act on the Hilbert space H^σ of the dimension d_σ. Note that all H^σ are of a finite dimension in our compact case. Annihilators of subgroups G_n in Σ are defined as $A_n = A(\Sigma, G_n) = (\sigma \in \Sigma : U_x^\sigma = I \text{ for all } x \in G_n)$.

For any additive complex measure μ on G and for any $\sigma \in \Sigma$ there exists a unique operator T_σ on H^σ such that $\langle T_\sigma \xi, \eta\rangle = \int_G \langle U_{x^{-1}}^\sigma \xi, \eta\rangle d\mu(x)$ for every $\xi, \eta \in H^\sigma$ (see [?]). Fourier-Stieltjes series of a measure μ is defined as

$$\sum_{\sigma\in\Sigma} d_\sigma \mathrm{tr}(T_\sigma U_x^\sigma)$$

(here and below $\mathrm{tr}(\cdot)$ denotes the trace of an operator).

We say that a formal series

$$S \sim \sum_{\sigma\in\Sigma} d_\sigma \mathrm{tr}(B_\sigma U_x^\sigma), \tag{1}$$

where B_σ are bounded linear operators on H^σ, is convergent to a function f at $x \in G$ if its partial sums

$$\sum_{\sigma\in A_n} d_\sigma \mathrm{tr}(B_\sigma U_x^\sigma)$$

are convergent to $f(x)$ at x.

We prove that *if a series* (**??**) *is everywhere convergent to a finite function f then f is integrable on G in the sense of some generalization of Henstock integral and* (**??**) *is the Fourier-Stieltjes series of the measure $\mu = \int f$.*

Some extensions to the non-abelian case of results of [?] related to the properties of the sets of uniqueness are also obtained.

Bibliography

[1] V. Skvortsov, F. Tulone, *Kurzweil-Henstock type integral on zero-dimensional group and some of its applications*, Czechoslovak Math. J. **58** (2008), 1167–1183.

[2] N. Kholshchevnikova, V. Skvortsov, *On U- and M-sets for series with respect to characters of compact zero-dimensional groups*, J. Math. Anal. Appl. **446** (2017), no. 1, 383–394.

[3] E. Hewitt and K. A. Ross, *Abstract Harmonic Analysis II*, New York, NY: Springer-Verlag New York, 1970.