

On variational and Riemann approach to some Lusin-type integrals

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Soon after discovering his renowned integral (equivalent to the narrow Denjoy integral, or *Denjoy–Perron integral*) [?], Ralph Henstock, in a remark in his textbook [?], suggested a similar Riemann-type approach that ought to lead to the other of Denjoy’s integral, now the wide Denjoy integral (or *Denjoy–Khintchine integral*). Since he provided there no detailed proof, this gave birth (much later) to a number of works on this problem [?, ?, ?]. The setting considered then was more general than Henstock’s, with some generalized continuity (of primitives) used instead of ordinary continuity (present in the wide Denjoy approach).

Being inspired by a recent exposition by Brian Thomson on measure-theoretic characterizations of ACG and VBG properties [?], in the present work we use Thomson’s weak and q -weak measures in characterizations of generalized wide Denjoy integrals, via absolute continuity of these measures. As a consequence, and a parallel result, we provide simplified Riemann definitions, equivalent to the definitions from [?, ?, ?].

Bibliography

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