On the generalized binomial transform

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Attaching an additional sequence $\{\alpha_n\}_{n\in\mathbb{N}_0}$ to the binomial transform, we obtain its extension that we use to define the generalized binomial transform $\mathcal{T}_\alpha$, which presents a correspondence between a set of infinitely continuously differentiable functions and a set of sequences linked to a generalized linear difference operator A_α. Taking different sequences $\{\alpha_n\}_{n\in\mathbb{N}_0}$ gives rise to a family of transforms $\mathcal{T}_\alpha$.

We make use of $\mathcal{T}_\alpha$ to map derivatives to A_α, and integrals to A_α^{-1} as well. The inverse transform $\mathcal{B}_\alpha$ of $\mathcal{T}_\alpha$ is introduced and its properties are studied. Choosing the sequence $\{\alpha_n\}_{n\in\mathbb{N}_0}$ such that $\alpha_n = (-1)^n$, it is shown that $\mathcal{B}_\alpha$ reduces to the Borel transform. Also, applying $\mathcal{T}_\alpha$ for the same sequence $\{\alpha_n\}_{n\in\mathbb{N}_0}$ to Bessel's differential operator $D_x = \frac{d}{dx}x\frac{d}{dx}$, we obtain discrete Bessel's operator $\Delta n\nabla$.

As applications of Bessel's differential and discrete operator, it is shown that eigenfunctions of Bessel's operator are mapped to eigenvectors of discrete Bessel's operator, and models of population growth are considered.
