Let \( \{f_n\} \) be an orthonormal system in a Hilbert space \( L_2(X) \). Then a set \( A \subset X \) is called a \( \mathcal{V}\)-set for the system \( \{f_n\} \) if convergence of a series \( \sum_n a_n f_n(x) \) to a finite summable function \( f \) on \( X \setminus A \) implies that this series is the Fourier series of \( f \). Setting \( f = 0 \) on \( X \setminus A \), we get the definition of a \( \mathcal{U}\)-set for the system \( \{f_n\} \). Each \( \mathcal{V}\)-set is evidently a \( \mathcal{U}\)-set.

We study analysis on Vilenkin groups \( G \), i.e., on zero-dimensional second-countable compact commutative groups (see \([?]\)). The elements of the dual group of \( G \) form an orthonormal system \( \{f_n\} \) in \( L_2(G) \).

Harris proved \([?]\) that any closed, measure zero subgroup of a Vilenkin group is a \( \mathcal{U}\)-set. Grubb found another examples of \( \mathcal{U}\)-sets and de-facto proved that any closed \( \mathcal{U}\)-set is a \( \mathcal{V}\)-set (see, for example, \([?, ?]\)). In \([?]\) some category properties of \( \mathcal{U}\)-sets are established.

In the multidimensional case, examples of countable \( \mathcal{U}\)-sets for square convergence are constructed in \([?]\). We introduce a multidimensional analog of Dirichlet sets in the product of Vilenkin groups and prove that all translations of such sets are \( \mathcal{V}\)-sets and therefore \( \mathcal{U}\)-sets. The full sequence of rectangular partial sums and restricted rectangular convergence are considered. The main tool of our investigation is quasi-measures and the concept of \( \Gamma\)-continuity of ones.

Bibliography


