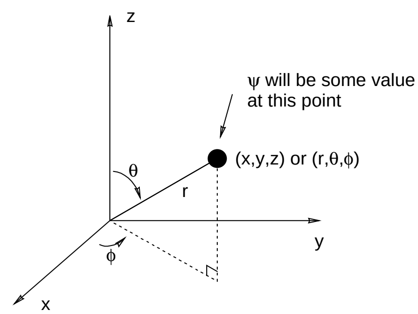
The solution to the Schroedinger equation for the hydrogen atom consists of two parts, a function Ψ (psi, pronounced “SIGH”), and an associated (total) enery, E:

$$\Psi_{nlm}(r,\theta,\varphi) = R_{nl}(r) \times \Theta_{lm}(\theta) \times \Phi_{m}(\varphi) \qquad r \textit{ radial or “altitude”}, \ \theta(\textit{theta}) \textit{ latitude, and } \varphi(\textit{phi}) \textit{ longitude}$$

$$R_{nl}(r) = (2/na_o)^{3/2}\left(\frac{(n-l-1)!(n+l)!}{2n}\right)^{1/2}(2r/na_o)^l e^{-r/na_o}\sum_{j=0}^{n-l-1}\frac{(-2r/na_o)^j}{j!(n-l-j-1)!(2l+j+1)!}$$

$$\Theta_{lm}(\theta) = 2^{-l-1/2}l![(2l+1)(l-|m|)!(l+|m|)!]^{1/2}|\sin^{|m|}\theta|\sum_{j=|m|}^{l}(-1)^{l-j}\frac{(1+\cos\theta)^{j-|m|}(1-\cos\theta)^{l-j}}{j!(l+|m|-j)!(l-j)!(j-|m|)!}$$

$$\Phi_m(\varphi) = (2\pi)^{-1/2}(\cos m\varphi + i\sin m\varphi)$$



$$E_n = \left(\frac{-1}{n^2}\right)\frac{(e_+^2)}{2a_o}$$

where

$$a_o = \frac{h^2}{4\pi^2 m_e(e_+^2)} = \textit{the “Bohr” radius}, \ 0.53\times 10^{-10}\, m$$

and

$$m_e = \textit{the mass of the electron}, \ 0.00091\times 10^{-27}\, kg$$

$$e_+ = \textit{the basic unit of charge}, \ 1.6\times 10^{-19}\, \textit{Coulombs}$$

Note that there is actually a whole set of “correct” solutions to the Schroedinger equation. To get any solution, replace n, l, and m with “reasonable” integers. For example, setting the parameters $n = 1$, $l = 0$, and $m = 0$, we get:

“altitude” part	latitude part	longitude part
$R_{10}(r) = \dfrac{2}{a_o^{3/2}}e^{-r/a_o}$	$\Theta_{00}(\theta) = \dfrac{1}{\sqrt{2}}$	$\Phi_0(\varphi) = \dfrac{1}{\sqrt{2\pi}}$

and

$$\Psi_{100} = \left(\frac{1}{\pi a_o^3}\right)^{1/2}e^{-r/a_o} \qquad E_1 = \frac{-(e_+)^2}{2a_o} = -2.17\times 10^{-18}\, \textit{Joules}$$

If you look carefully, the "rules" we learn for *n*, *l*, and *m* are present in the solution. Look for places, for example, where *n* is in the denominator, as 1/*n*. How does that restrict the possible values for *n*? Similarly, there are several factorials in this equation, such as $(n + l)!$. That means the product of all the integers from 1 to $n+l$. Factorials can only be taken of nonnegative integers. (0! is defined to be 1, but (–1)! is meaningless.) The presence of certain factorial terms in the solution of the Schroedinger equation places strict restraints on the values of *n*, *l*, and *m*. Can you find those terms?